

# Advanced GARCH Modeling Techniques and Risk-Return Relationship in the Vietnamese Stock Market

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## Abstract

This study investigates the Vietnamese stock market volatility focusing on the VN-INDEX from the Ho Chi Minh Stock Exchange, during the period 2004-2024. Both symmetric and asymmetric GARCH models are implemented with three different error term distribution assumptions to analyse the volatility persistence and clustering as well as any leverage effects on stock returns. The best fitting model for the dataset seems to be the EGARCH (2,1) using a Skew-t distribution assumption. The positive risk-return link is confirmed, with higher volatility associated with greater expected returns. This is one of the very few studies focused on a long period that moreover includes the effects that some unexpected exogenous shocks (like the Covid-19 pandemic and other recent geopolitical events) could generate on the

parameters estimates. Thus, it can encompass different market phases, including years of economic growth, stability, volatility, and downturns, thus offering a comprehensive view of the Vietnamese market's behaviour under dissimilar economic and financial conditions. This research offers valuable insights into the nature of uncertainty in the Vietnamese stock market, helping investors in their decision-making processes and contributing to the overall understanding of the market. Indeed, understanding volatility and its implications is essential for investors, policymakers, and researchers, particularly in emerging markets like Vietnam, where financial systems are evolving rapidly.

**Keywords:** stock market volatility; risk-return relationship; GARCH models; error term distribution; emerging markets.

*JEL-code:* G1; G15; G17; C1; C58.

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## 1. Introduction

As is well known, central to the discussion of financial markets are both the concept of volatility, typically seen as a measure of the total risk and uncertainty associated with financial assets (Ahmad *et al.*, 2016), as well as the volatility-return relationship, as a particular declination of one of the basic fundamental principles in finance.

While the last link has been extensively studied, since many decades, in the context of developed capital markets, there are fewer papers on the emerging ones, as happened for the Vietnamese stock exchange.

With more detail, referring to developed stock markets, many empirical studies examined returns and the behavior of volatility producing contrasting results; for a complete review of this topic it is possible to consider the contribution of Fu (Fu, 2009). Indeed, in relation to the emerging markets, it is possible to refer to Yavas and Dedi (Yavas & Dedi, 2016) to understand that even in such financial realities are reached non-unanimous conclusions. Within the Vietnamese context, there are some studies that focused on stock market index symmetric models (Nguyen & Le, 2010; Luu & Hoang, 2024), and only few other contributions testing more sophisticated approaches (Ho *et al.* 2017; Nguyen & Darné, 2018; Nguyen & Nguyen, 2019; Tran, 2022). This fact clearly highlights the need for further, more in-depth, researches to enhance the understanding and management of financial market risks in Vietnam.

Stating the above, this study models volatility and the risk-return relationship of the VN-INDEX on Ho Chi Minh Stock Exchange in the period from August 2004 to August 2024 via both symmetric and asymmetric GARCH family models with three distributional assumptions (Normal, Student-t and Skew-t) to assess which of them is the better performing. To the best of our knowledge, this is one of the very few studies (Nguyen & Nguyen, 2019; Tran, 2022; Hoang & Luu, 2024) focused on a so long period. Moreover, it refers to the most recent data and, for this reason, it can also somewhat include the effects that some unexpected exogenous shocks (like the Covid-19 pandemic as well as other recent geopolitical events) could generate on the parameters estimates of the models used to analyse the relationship between volatility and expected returns in the Vietnamese market. As a consequence, it can encompass different market phases, including years of economic growth, stability, volatility, and downturns, thus offering a comprehensive view of the Vietnamese market's behavior under dissimilar economic and financial conditions.

The main key objectives of this study include understanding the behavior of Vietnam's stock return volatility over time, identifying the GARCH model specification that best describe the VN-INDEX dataset using the most suitable error distributional assumptions, and assessing if there is a statistically significant relationship between volatility and expected returns in the analysed market.

This research will offer valuable insights into the nature of uncertainty in the Vietnamese stock market, helping investors in their decision-making processes and contributing to the overall understanding of the market. Indeed, understanding volatility and its implications is

essential for investors, policymakers, and researchers, particularly in emerging markets like Vietnam, where financial systems are evolving rapidly.

The paper is organised as follows: Section 2 provides a comprehensive overview of the theoretical and empirical literature on stock return volatility and the risk-return relationship, particularly focusing on the implementation of GARCH models in different market contexts including Vietnam; Section 3 illustrates the methodology used, while Section 4 presents and discusses the results obtained. Section 5 concludes.

## **2. Literature Review**

This section offers a review of the studies that used some sophisticated approaches in analysing returns and the behavior of volatility. The aim is to provide, at the same time, some insights into practical applications, evidences observed empirically in different market contexts and to highlight gaps in analyses that can still be filled with reference to the emerging markets and, in particular, in the Vietnamese context.

Actually, existing literature reveals conflicting results regarding the risk-return relationship (Fu, 2009; Yavas & Dedi, 2016). In a nutshell, these discrepancies arise, not only from the consideration of different financial markets and countries, but also from variations in data frequency, sample periods and model specifications.

Poon and Taylor (Poon & Taylor, 1992) observed a weak link between risk and return in developed markets. Their research investigated the connection between stock returns and volatility in the UK by analyzing daily, weekly, fortnightly, and monthly returns on the Financial All index. Employing monthly variances and ARCH models, they found that expected returns had a positive, albeit not statistically significant, relationship with anticipated volatility. Additionally, evidence of a negative relationship was identified when volatility expectations were measured using the standard deviation.

Theodossiou and Lee (Theodossiou & Lee, 1995) examined stock market volatility and its relationship to expected returns in ten industrialized countries (Australia, Belgium, Canada, France, Italy, Japan, Switzerland, Germany, United Kingdom and United States). Using the GARCH-M model, they tested three specifications (linear, square root, and log-linear) for the relationship between conditional variance and expected market returns. Their findings aligned with some previous studies, indicated no significant relationship between conditional volatility and expected returns.

De Santis and Imrohorglu (De Santis & Imrohorglu 1997) observed that in emerging markets, volatility clustering showed similarities to developed markets, with high levels of persistence. They found that using a fat-tailed distribution improves model fit and noted no risk premium for market-wide risk for Asian countries included in their analysis. However, they observed a stronger risk-return relationship in emerging markets. Their research investigated how stock returns and volatility behaviors fluctuated over time across several emerging markets, focusing on the variations in stock return volatility, using GARCH-M(1,1) model with two measures of risk (conditional variance and conditional standard deviation).

These findings appear to challenge the conventional asset pricing theories that propose a built-in risk premium in equity returns.

Xing & Howe (Xing & Howe, 2003) argued that the global market factor is crucial when evaluating the risk-return relationship in a partially integrated market. By utilizing a bivariate GARCH-M model on weekly stock index returns from the UK and the global market, they found a significant positive relationship between stock returns and return variance in the UK market, after accounting for the covariance between UK and global market returns.

Menggen (Menggen, 2007) examined the risk-return trade-off in the Chinese emerging stock markets of Shanghai and Shenzhen. A positive and statistically significant risk-return relationship was observed only in the daily returns of the Shenzhen Stock Exchange, while this relationship became not significant for lower frequency returns. Conversely, in the Shanghai Stock Exchange, the conditional mean of stock returns was generally negatively but not significantly related to its conditional variance, except for a positive yet not significant relationship in the GARCH-M model for daily returns. The variance of stock returns typically showed a significant and persistent effect, regardless of data frequency. Additionally, the asymmetric effect was highly significant for daily returns in both markets, but weakened or disappeared for lower frequency data.

John and Oduro (John & Oduro, 2016) analyzed the volatility and risk-return relationship of some stocks on the Ghana Stock Exchange using the GARCH-M(1,1) model with three different distributional assumptions: Student-t, GED (Generalized Error Distribution) and Gaussian. They found that all stocks were highly volatile. Their results revealed a positive risk premium, indicating that investors were compensated for holding riskier assets. Additionally, the asymmetric models provided a better fit than the symmetric one, suggesting the presence of a leverage effect in the selected stocks. The TGARCH-M(1,1) model with the Student-t distribution was identified as the most suitable.

Drachal (Drachal, 2017) examined volatility, leverage effects, and the risk-return trade-off in weekly returns across emerging markets in Central and Eastern Europe. Using GARCH-M and different asymmetric models (T-GARCH, EGARCH, GJR-GARCH, and APARCH), the study identified significant leverage effects in Romania, Hungary, Czech Republic, Russia and Poland. Additionally, a significant negative risk-return trade-off was observed in Bulgaria, Latvia, Lithuania and Montenegro, while Estonia exhibited a significant positive one.

Another study (Herbert *et al.*, 2019) examined the phenomenon of volatility clustering, leverage effect (asymmetry) and the risk-return trade-off in daily stock returns of the Nigerian stock market using both GARCH(1,1) and GJR-GARCH(1,1) models. Their findings confirmed the presence of persistent volatility clustering and significant leverage effects. Consistent with the risk-return trade-off, they found that investors demand higher returns for higher risk investments, in high risk firms or in inefficient and unstable markets.

Dwarika *et al.* (2021) explored the volatility dynamics and the risk-return relationship in the South African market by examining FTSE/JSE All Share Index returns. They used some GARCH models with different probability distributions for the model's innovations and

discovered persistent high levels of volatility and a significant positive risk-return relationship in the South African market.

Gazali *et al.* (2022) analysed the risk-return trade-off and volatility behavior in daily and weekly stock returns in the Indonesian stock market using the GARCH-M model with symmetric GARCH(1,1). The study's primary finding was that in the Indonesian market, both the stock index and individual stocks exhibit time-varying volatilities. However, a positive risk-return relationship, as suggested by the classical investment theory, was found only in the stock index and not in all individual stocks.

The literature on the volatility of stock returns in Vietnam and the risk-return relationship, particularly employing advanced models, is scarce.

Nguyen and Le (Nguyen & Le, 2010) employed a GARCH model to analyse volatility in two Vietnam's stock markets showing the presence of a GARCH effect. Additionally, their research assessed some factors influencing stock return volatility, including price margin policies, trading volume, and the leverage effect.

Some researchers (Man & Anh, 2013; Phuoc *et al.*, 2017) conducted studies on the Value at Risk (VaR) model, incorporating the use of ARCH and GARCH models to estimate variance parameters. The results revealed that the GARCH model is a useful tool in Vietnamese risk management.

Ho *et al.*, (Ho *et al.*, 2017) empirically investigate the volatility pattern of daily returns in Vietnam stock market during the period 2005-2016, using GARCH(1,1), EGARCH(1,1) and TGARCH(1,1) models. The study also provides evidence of the existence of asymmetric effects (leverage) via the parameters of the EGARCH(1,1) that show that negative shocks have significant impacts on conditional variance (fluctuation).

In a successive working paper (Nguyen & Darné, 2018), GARCH-type models are applied to the VN index and HNX index from the Ho Chi Minh Stock Exchange and Hanoi Stock Exchange for the period 2007-2015. Empirical findings indicated that the FIAPARCH model is the most appropriate for both indexes.

Nguyen and Nguyen (Nguyen & Nguyen, 2019) applied both symmetric (GARCH, GARCH-M) and asymmetric (EGARCH and TGARCH) models to analyse the Ho Chi Minh Stock Exchange stock price volatility in the period 2001-2019. The study found that GARCH(1,1) and EGARCH(1,1) are the most suitable models for measuring both symmetric and asymmetric volatility of the VN-Index. It also confirmed the existence of asymmetric effects through the TGARCH(1,1) model, showing that positive shocks significantly impact volatility. These results suggest that GARCH models are effective tools for risk forecasting.

In his paper, Tran (Tran, 2022) analysed the statistical characteristics of the stock price series and its relationship with the financial cycle in the period 2000-2020, selecting in relation to the data set used the most adequate GARCH model and its best errors distribution. As a result, EGARCH with Student-t statistic distribution seemed to be the most appropriate for illustrating stock price movements and return volatility. Despite the obtained results, the same

author suggested the need for further research on the topic.

A very recent paper (Hoang & Luu, 2024) studied the volatility dynamics and identified regime-switching behaviours in the Vietnam - Ho Chi Minh Stock Index in the period 2012-2023, employing different traditional GARCH family models. The study finds that index returns exhibit common characteristics like fat tails, long-memory volatility, clustering behaviors, and leverage effects. Market dynamics vary based on the time horizon, with single-regime models performing better for short-term volatility forecasting. However, for medium and long-term horizons, regime-switching models are more effective, as they adapt to changing market conditions, transitioning between low- and high-volatility states.

Despite these literature contributions - which actually cover very different topics - research on returns volatility and the risk-return relationship in Vietnam still remains limited. Furthermore, no paper has yet highlighted the effects that some unexpected exogenous shocks (like the Covid-19 pandemic as well as other recent geopolitical events) could generate on the parameters estimates of the models used to assess the actual relationship between volatility and expected returns in the Vietnamese market. This circumstance creates a gap in the literature that underlines the need for further studies.

### 3. Method

As anticipated in Section 1, this paper tries to model volatility and the risk-return relationship of the VN-INDEX on Ho Chi Minh Stock Exchange, analysing the behavior of both symmetric and asymmetric GARCH family models with three different distributional error term assumptions (Normal, Student-t and Skew-t) with the aim of assessing which of them performs best.

For this purpose, the research methodology will be structured as follows. First, all the usual preliminary analyses will be conducted to state the suitability of the returns' time series analysis, thus assessing their stationarity, normality, autocorrelation and heteroscedasticity. In more details, (i) time series plots, Augmented Dickey-Fuller (ADF - Dickey & Fuller, 1981) and Phillips-Perron (PP - Phillips & Perron, 1988) tests will be used for stationarity (Liu *et al.*, 2020); (ii) skewness and kurtosis measures, Quantile-Quantile (Q-Q) Plot (Ghasemi & Zahediasl, 2012), histogram of returns with kernel and normal fit (González-Rivera *et al.*, 2008), Shapiro-Wilk (Shapiro & Wilk, 1965) and Jarque-Bera (Bera & Jarque, 1981) tests will be considered in assessing the degree of normality; (iii) Autocorrelation Function plot (ACF - Box & Jenkins, 1976) and Ljung and Box (LB - Ljung & Box, 1978) test will be implemented to examine autocorrelation, and (iv) Autoregressive Conditional Heteroskedastic Lagrange Multiplier (ARCH-LM) test will be used (in line with Engle, 1982 and John & Oduro, 2016) to support whether the VN-INDEX data have ARCH effect. These procedures align with the standard methods described by Khan *et al.*, 2016; Liu, 2019; Dwarika *et al.*, 2021 and Bai, 2022.

Following these preliminary analyses, both symmetric and asymmetric GARCH family models will be estimated to capture the volatility dynamics of VN-INDEX.

These include:

- the Autoregressive Conditional Heteroskedasticity (ARCH(q) - Engle, 1982) model

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 \quad \text{with} \quad \alpha_0 > 0 \quad \alpha_i \geq 0 \quad \forall i \quad [1]$$

where  $\sigma_t^2$  is the conditional variance and  $\varepsilon_t = \sigma_t z_t$  (with  $z_t \sim i.i.d.N(0,1)$ ) is the error term. For the model to be well-defined, all coefficients must be positive, and their sum should be less than one;

- the Generalized Autoregressive Conditional Heteroskedasticity (GARCh(p,q) - Bollerslev, 1986) model, where the conditional variance is defined as

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2$$

with:  $\alpha_0 > 0 \quad \alpha_i, \beta_j \geq 0$

[2]

where, as in [1]  $\sigma_t^2$  is the conditional variance,  $\alpha_0$  (baseline volatility) is constant,  $\varepsilon_t = \sigma_t z_t$  (with  $z_t \sim i.i.d.N(0,1)$ ) is the error term, and  $\alpha_i$  captures the effect of past shocks ( $\varepsilon_{t-i}^2$ ),  $q$  are the prediction error delays, while the new parameter  $\beta_j$  represents the dependence of the current variance on the past one ( $\sigma_{t-j}^2$ ),  $p$  representing the delays relating to past values of variance. In more detail,  $\alpha_i$  measures the immediate impact of new information (or shocks) on the current volatility, while  $\beta_j$  describes the persistence of volatility over time, which means how much of the past period's variance carries over to the current period. To avoid variance explosions (condition of stability)  $\sum_{i=1}^q \alpha_i + \sum_{j=1}^p \beta_j < 1$ .

- the Exponential Generalized Autoregressive Conditional Heteroskedasticity (EGARCH(p,q) - Nelson & Cao, 1992) model

$$\log(\sigma_t^2) = \alpha_0 + \sum_{j=1}^p \beta_j \log(\sigma_{t-j}^2) + \sum_{i=1}^q \left[ \alpha_i \frac{|\varepsilon_{t-i}|}{\sigma_{t-i}} + \gamma_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right]$$

[3]

where, as in previous model,  $\sigma_t^2$  is the conditional variance,  $\alpha_0$  is the constant term,  $\varepsilon_t = \sigma_t z_t$  (with  $z_t \sim i.i.d.N(0,1)$ ) is the error term,  $\alpha_i$  measures the effect of past shocks on volatility,  $\beta_j$  quantifies the persistence of past volatility, while a new parameter,  $\gamma_i$ , captures asymmetry (*leverage effect*). Unlike the previous model, in this case  $\alpha_i$  and  $\beta_j$  are unconstrained and the condition of stationarity to refer to is  $\sum_{j=1}^p \beta_j < 1$ . With more details, model [3] defines the logarithm of volatility to ensure

non-negativity and assesses a leverage effect via asymmetry; in particular when  $\gamma_i < 0$ , negative shocks increase volatility more than the positive ones.

- the Glosten-Jagannathan-Runkle GARCH (GJR-GARCH(p,q) - Glosten *et al.*, 1993) model

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q (\alpha_i + \gamma_i I_{t-1}) \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2$$

$$\alpha_0 > 0, \alpha_i \geq 0, \beta_j \geq 0, \alpha_i + \gamma_i \geq 0$$

$$I_{t-i} = 0 \text{ if } \varepsilon_{t-i} \geq 0 \text{ and } I_{t-i} = 1 \text{ if } \varepsilon_{t-i} < 0$$

[4]

where, similarly to the previous two models,  $\sigma_t^2$  is the conditional variance,  $\alpha_0$  is a constant term,  $\varepsilon_t = \sigma_t z_t$  (with  $z_t \sim i.i.d.N(0,1)$ ) is the error term,  $\alpha_i$  measures the effect of past shocks on volatility,  $\beta_j$  measures the past volatility persistence,  $\gamma_i$  catches the leverage effect via asymmetry, while the new term  $I_{t-i}$  is an indicator variable. In this case the condition of stationarity to refer to is  $\sum_{i=1}^q (\alpha_i + \gamma_i/2) + \sum_{j=1}^p \beta_j < 1$ . In particular, when  $\gamma_i < 0$ , negative shocks increase volatility less than the positive ones; actually, this is an atypical phenomenon compared to the usual behavior of financial markets.

- the Asymmetric Power ARCH (APARCH(p,q) - Ding *et al.*, 1993) model

$$\sigma_t^\delta = \alpha_0 + \sum_{i=1}^q \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^p \beta_j \sigma_{t-j}^\delta$$

[5]

where, as in the previous models,  $\sigma_t^2$  is the conditional variance,  $\alpha_0 > 0$  is a constant term,  $\varepsilon_t = \sigma_t z_t$  (with  $z_t \sim i.i.d.N(0,1)$ ) is the error term,  $\alpha_i \geq 0$  captures the past shocks effect,  $\beta_j \geq 0$  quantifies the past volatility persistence,  $\gamma_i$  refers to asymmetry. In particular, if  $\gamma_i < 0$ , negative shocks increase volatility more than positive ones; the new parameter  $\delta > 0$  is a *power parameter* that transforms volatility; in particular, if  $\delta = 2$  the classical GARCH model is obtained).

Indeed, each of them offers unique features that allow for a more detailed analysis of volatility, including the ability to capture asymmetries and leverage effects in the return series (Nelson, 1991; Glosten *et al.*, 1993; Ding *et al.*, 1993).

In particular, the models [2] - [5] will be tested using three different assumptions for the error term distribution, namely, the Normal, the Student-t and the Skew-t; this, exactly to capture the market's complex volatility patterns.

The best-fitting model will be identified by comparing the AIC (Akaike Information Criterion - Akaike, 1974) and BIC (Bayesian Information Criterion - Schwarz, 1978) values and

performing an additional diagnostic check (Minović, 2008) to ensure the adequacy in capturing market volatility (Sun & Zhou, 2014). Indeed, these models will be evaluated based on their ability to eliminate autocorrelation and ARCH effects in standardised residuals, as well as the adequacy of the error distributional assumption. With more detail, following the approach generally proposed by researchers (Xing & Howe, 2003; Menggen, 2007; Ho *et al.*, 2017; Dwarika *et al.*, 2021; Scherer Perlin *et al.*, 2021 and Bai, 2022), the best model will be individuated considering the lowest AIC and BIC values and a subsequent validation of the choice via an examination of the selected GARCH models' standard residuals to assess whether the risk has been adequately captured.

The results provided by the selected model will finally allow the risk-return relationship to be investigated, testing the hypothesis if higher volatility is associated with higher expected returns in the Vietnamese context. This will be done via the following GARCH-in-Mean (GARCH-M) and Exponential GARCH-in-Mean (EGARCH-M) models (Engle *et al.*, 1987; Nelson, 1991):

- the Generalized Autoregressive Conditional Heteroskedasticity-in-Mean (GARCH-M(p,q)) model (Engle, Lilien and Robins, 1987)

$$y_t = \mu + \sum_{j=1}^p \lambda_j \sigma_{t-j}^x + \varepsilon_t, \varepsilon_t \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2$$

[6]

where  $y_t$  is an observed value modeled as a conditional mean ( $y_t$  is an asset return;  $y_t$  is the time series equation with the volatility term in the mean),  $\mu$  is a constant,  $x = 1$  or  $2$ ,  $\varepsilon_t$  is the error term that follows - as in all the other previous models - a normal distribution, the conditional volatility ( $\sigma_t^2$ ) follows a GARCH process, while the  $\lambda_j$  (risk premium parameter) is a new coefficient that measures the impact of conditional variance on the mean. In other words, it quantifies the effect of volatility on returns ( $\lambda_j > 0$  means higher risk that implies higher expected return).

- the Exponential GARCH-in-Mean (EGARCH-M(p,q)) model. Actually, it is an extension of both the GARCH and the GARCH-M models and was developed to capture the asymmetric effects in the volatility process. This makes it possible to model situations in which volatility itself directly affects returns. The model consists of two main equations, that is (i) the mean equation ( $y_t$ ) that has the same structure seen in the standard GARCH-M model (and thus incorporates a volatility-dependent term) and (ii) the conditional variance equation ( $\log(\sigma_t^2)$ ) that follows the EGARCH(p, q) process. Thus, the EGARCH-M(p,q) model can be written as

$$y_t = \mu + \sum_{j=1}^p \lambda_j \sigma_{t-j}^x + \varepsilon_t, \varepsilon_t \sim N(0, \sigma_t^2)$$

$$\log(\sigma_t^2) = \alpha_0 + \sum_{j=1}^p \beta_j \log(\sigma_{t-j}^2) + \sum_{i=1}^q \left[ \alpha_i \frac{|\varepsilon_{t-i}|}{\sigma_{t-i}} + \gamma_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right]$$

[7]

where, as in the other previous models,  $\alpha_0$  is a constant,  $\alpha_i$  measures the effect of the past shocks size on current volatility,  $\beta_j$  represents the persistence of volatility over time and  $\gamma_i$  captures the shocks' asymmetry, discriminating between positive and negative effects, making it possible to test whether and how expected volatility directly influences returns. Again, the  $\gamma_i$  term allows the leverage effect to be modelled, i.e. the tendency of financial markets to have more pronounced increases in volatility in response to negative returns (if  $\gamma_i < 0$ , negative shocks increase volatility more than positive shocks, while if  $\gamma_i > 0$  negative shocks increase volatility less than positive shocks).

Both forms of the GARCH-in-Mean model, incorporating either conditional variance or conditional standard deviation in the mean equation, will be estimated. Then, also in this case, the AIC and BIC will be used to assess which specification fits better the data (De Santis & Imrohroglu, 1997; French *et al.*, 1987; Baillie & De Gennaro, 1990).

#### 4. Empirical Results

This study utilises historical daily closing price data of the Vietnam Ho Chi Minh stock index (VN-INDEX) over a 20-year period, from August 1, 2004 to August 31, 2024, obtained from the Refinitiv LSEG database. The data consists of adjusted closing prices, with a total of 2744 time series observations (Note 1).

Actually, this section presents both graphical and numerical analyses. First, a basic analysis of the VN-INDEX data is shown, followed by a description of its returns' characteristics through preliminary tests. Subsequently, the focus shifts to the volatility dynamics within the Vietnamese market, analysed using GARCH-type models. Finally, the most fitting GARCH model for capturing risk and data dynamics is identified to explore the risk-return relationship and the pricing of risk in the Vietnamese market. Figure 1 shows the dynamics of VN-INDEX price data over the time frame analysed.



Figure 1. VN-INDEX's price dynamics (2004-2024).

Initial tests of the VN-INDEX return series have shown its stationarity, suggesting the series is suitable for more complex analyses. Basic descriptive statistics and tests of normality revealed that the index returns do not follow a normal distribution, displaying an asymmetric, left-skewed nature, with heavier tails than a normal distribution. Additionally, the analyses of ACF plot and LB test have shown that the log returns of the VN-INDEX lack autocorrelation. However, analyses have confirmed the presence of heteroskedasticity and an ARCH effect within the return series, underlining the necessity for models able to capture such time-variant volatility, like GARCH models (Note 2). These findings are consistent with the expected risks and the imbalanced nature of financial returns. The data also have shown volatility clustering phenomenon, which further supports using the GARCH model for a deeper examination of the market dynamics.

A variety of GARCH (1,1) type models have been used as tests on the Vietnamese market's return dynamics. These models aimed to verify specific behavioral patterns within the market, such as volatility clustering, the presence of heavy-tailed distributions, and asymmetrical effects (leverage effects).

An initial analysis of the volatility patterns in the Vietnamese market has been undertaken via the implementation of a basic GARCH(1,1) model, then more sophisticated asymmetrical GARCH-type models. As it is well known, the structuring of a GARCH model begins by selecting the number of past observations, or lags, the variance equation, and parameters distribution. Thus, three versions of a GARCH model have been considered; indeed, each of them uses a different equation to calculate volatility, but maintains a consistent number of lags and the same underlying assumptions about the distribution of residuals. As usual, in the mean equation a constant term has been considered, while each term in the volatility equation referred to lag-1. The enhanced models included the GJR-GARCH(1,1), EGARCH(1,1), and APARCH(1,1) and had model specifications to detect additional features of the market such

as asymmetric volatility and the leverage effect.

Tables 1 to 3 display findings from the application of both symmetric and asymmetric GARCH(1,1) models, which incorporate three distinct distributions for the error term, including Normal, Student-t and Skewed-t distribution.

Table 1. GARCH models results: Normal distribution assumption for the error term.

	<b>GARCH (1,1)</b>	<b>EGARCH(1,1)</b>	<b>GJR-GARCH (1,1)</b>	<b>APARCH (1,1)</b>
<b>Mean</b>				
$\mu$ (constant)	0.000* (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
<b>Variance</b>				
$\omega$ (constant)	0.000* (0.000)	-0.361*** (0.055)	0.000*** (0.000)	0.000 (0.000)
$\alpha_1$ (ARCH effect)	0.101*** (0.010)	-0.063*** (0.012)	0.063*** (0.006)	0.071*** (0.010)
$\beta_1$ (GARCH effect)	0.876*** (0.011)	0.960*** (0.006)	0.870*** (0.008)	0.871*** (0.013)
$\gamma$ (leverage effect)		-0.201*** (0.017)	0.063*** (0.012)	-0.086* (0.039)
$\delta$				2.684*** (0.049)
$\alpha_1 + \beta_1$	0.977	0.897	0.933	0.942
Log likelihood	8.734.322	8.753.612	8.742.133	8.731.007
Akaike Inf. Criterion (AIC)	-6.362	-6.376	-6.367	-6.359
Bayesian Inf. Criterion (BIC)	-6.352	-6.363	-6.355	-6.344

Note 1: \*\*\*Significant at 0.1%, \*\* Significant at 1%, \*Significant at 5%

Note 2: The values in the parenthesis represent the standard errors

Table 2. GARCH models results: Student-t distribution assumption for the error term.

	<b>GARCH (1,1)</b>	<b>EGARCH(1,1)</b>	<b>GJR-GARCH (1,1)</b>	<b>APARCH (1,1)</b>
<b>Mean</b>				
$\mu$ (constant)	0.001*** (0.000)	0.001*** (0.000)	0.001*** (0.000)	0.001*** (0.000)
<b>Variance</b>				
$\omega$ (constant)	0.000* (0.000)	-0.515*** (0.055)	0.000*** (0.000)	0.000 (0.000)
$\alpha_1$ (ARCH effect)	0.144*** (0.019)	-0.093*** (0.016)	0.071*** (0.009)	0.101*** (0.028)
$\beta_1$ (GARCH effect)	0.829*** (0.021)	0.944*** (0.006)	0.789*** (0.015)	0.791*** (0.045)
$\gamma$ (leverage effect)		-0.258*** (0.026)	0.150*** (0.028)	-0.168** (0.059)
$\delta$				2.827*** (0.064)
$\alpha_1 + \beta_1$	0,973	0,851	0,860	0,892
Log likelihood	8.793.654	8.811.580	8.804.655	8.794.911
Akaike Inf. Criterion (AIC)	-6.406	-6.418	-6.413	-6.405
Bayesian Inf. Criterion (BIC)	-6.395	-6.405	-6.400	-6.390

Note 1: \*\*\*Significant at 0.1%, \*\* Significant at 1%, \*Significant at 5%

Note 2: The values in the parenthesis represent the standard errors

Table 3. GARCH models results: Skewed-t distribution assumption for the error term.

	GARCH (1,1)	EGARCH(1,1)	GJR-GARCH (1,1)	APARCH (1,1)
<b>Mean</b>				
$\mu$ (constant)	0.001*** (0.000)	0.001*** (0.000)	0.001*** (0.000)	0.001*** (0.000)
<b>Variance</b>				
$\omega$ (constant)	0.000* (0.000)	-0.484*** (0.041)	0.000*** (0.000)	0.000 (0.000)
$\alpha_1$ (ARCH effect)	0.129*** (0.015)	-0.084*** (0.015)	0.071*** (0.008)	0.079*** (0.021)
$\beta_1$ (GARCH effect)	0.841*** (0.018)	0.947*** (0.004)	0.815*** (0.013)	0.804*** (0.089)
$\gamma$ (leverage effect)		-0.244*** (0.023)	0.116*** (0.023)	-0.122* (0.085)
$\delta$				2.902*** (0.141)
$\alpha_1 + \beta_1$	0,970	0,863	0,886	0,883
Log likelihood	8.817.701	8.834.336	8.827.432	8.816.420
Akaike Inf. Criterion (AIC)	-6.423	-6.434	-6.429	-6.420
Bayesian Inf. Criterion (BIC)	-6.410	-6.419	-6.414	-6.403

Note 1: \*\*\*Significant at 0.1%, \*\* Significant at 1%, \*Significant at 5%

Note 2: The values in the parenthesis represent the standard errors

As known, the  $\beta_1$  parameter (GARCH effect) indicates how long the effects of a volatility shock may last (a higher  $\beta_1$  suggests these effects could persist for a longer period). On the other hand,  $\alpha_1$  (ARCH effect) reflects how quickly volatility reacts to fresh market information (larger  $\alpha_1$  values suggest higher volatility in response to recent market movements), which can lead to more unpredictable volatility patterns. The results show that both  $\alpha_1$  and  $\beta_1$  are strongly statistically significant ( $p < 0,01$ ) for all the GARCH models implemented. Moreover, the conditional variance equation reveals that volatility is predominantly influenced by the GARCH effect, more than the ARCH one, across all the models, thus suggesting that when shocks occur, they have a lasting impact on the VN-Index's volatility.

Most models exhibit a positive  $\alpha_1$ , implying that new market information seems to have a significant and immediate impact on volatility, but not overwhelmingly strong. However, the EGARCH model shows a significant and negative  $\alpha_1$ , suggesting that positive market shocks tend to decrease volatility, while, vice-versa, negative shocks tend to increase it. Actually, the different sign of  $\alpha_1$  in the EGARCH could be a consequence of the fact that this kind of model can handle asymmetric effects of shocks.

In all the implemented models,  $\beta_1$  is positive, signaling that volatility tends to have a lasting effect.

Volatility can be also characterized by adding the ARCH and GARCH effects together ( $\alpha_1 + \beta_1$ ), with all models showing a combined effect slightly below one, not only fulfilling the condition for long-term variance to take a positive and finite value, but also affirming the presence and persistence of volatility clustering.

The  $\gamma$  values reveal the presence of an asymmetric (leverage) effect. In the EGARCH and GJR-GARCH models  $\gamma$  is, respectively, negative and positive, although statistically significant for all three distribution assumptions, indicating the presence of asymmetric volatility, meaning that the shocks - whether positive or negative - can have different impact on volatility. This result implies that negative shocks have a greater impact on volatility than positive ones. In other words, negative market news tends to increase market volatility more than positive news of the same magnitude decrease it; in this way, the leverage effect in the VN-INDEX is demonstrated. Actually, this finding is in line with Ho *et al.*, (2017). The negative value of  $\gamma$  in APARCH(1,1) for all the three distributions further supports this conclusion.

As known, in the APARCH model,  $\delta$  is used to capture the power to which the absolute value of the lagged error term is raised; the threshold  $\delta = 2$  indicates a symmetric response of volatility to positive and negative shocks, since squaring the lagged error term (as in a standard GARCH model) does not differentiate between the direction of the shock. A  $\delta > 2$  (as in the results obtained) implies that larger changes in the stock prices (either upward or downward) have a disproportionately higher impact on the volatility of the index. This could be interpreted as the market being highly sensitive to significant news or events; in more detail, the market might remain relatively stable in response to small day-to-day fluctuations but could exhibit considerable volatility in response to more substantial changes, which could be due to major economic news, policy changes, or other significant events.

In summary, the parameter estimates highlight the persistence of long-term volatility, along with other characteristics such as volatility clustering, asymmetric volatility and leverage effects, in the VN-INDEX returns.

Considering the suitability of all the GARCH models discussed above, it is necessary to choose, among them, the one able to capture in the best way the risk and return dynamics from the VN-INDEX data. This can be achieved by comparing some fit quality metrics, namely the Akaike and the Bayesian Information Criteria (AIC and BIC). In particular, following the approach of Scherer Perlin *et al.* (Scherer Perlin *et al.*, 2021), the AIC and BIC values for different GARCH models have been calculated and the best model chosen on the basis of the lowest Information Criteria values.

According to AIC, the best model specification is ARMA(1,0) - EGARCH(2,1), while BIC suggests ARMA(0,0) - EGARCH(1,1) (Note 3). Both criteria agree that Skewed-t distribution is the best fitting error distribution. Stating the above, to decide which model between the aforementioned fits better the VN-INDEX volatility dynamics, a diagnostic check was

conducted on their standardised residuals. In particular, standard deviation specifications, Q-Q plots and other normality tests, Auto Correlation Function - ACF-plot and Ljung-Box (LB) test, together with ARCH effects (Ljung-Box - LB2 - and Autoregressive Conditional Heteroskedastic Lagrange Multiplier - ARCH-LM - tests) have been employed (Note 4).

In summary, the standardised residuals follow an asymmetric and non-normal distribution. Both models adequately captured the volatile nature of the innovation, since heteroskedasticity is confirmed to be present. However, the absence of autocorrelation can only be observed in the results for ARMA(1,0)-EGARCH(2,1), implying this model is better specified for the return series. Actually, the result of the EGARCH being the best fitting model is in line with Ho *et al.* (Ho *et al.*, 2017) as well as Dwarika (Dwarika, 2023) and Bai (Bai, 2022) in relation to other countries.

To address the question whether there is a risk-return trade off in Vietnamese stock market, two versions of asymmetric EGARCH(2,1)-M model have been used: in the first, the conditional standard deviation as a potential explanatory variable for expected returns was adopted while, in the second, it was replaced with the conditional variance. In both cases a Skewed-t distribution assumption was assumed.

Table 4. Estimates results for EGARCH (2,1)-M.

	Conditional standard deviation used in the mean equation	Conditional variance used in the mean equation
<b>Mean</b>		
$\mu$ (constant)	0.011* (0.002)	0.012* (0.003)
AR1	0.058*** (0.017)	0.064** (0.020)
$\lambda$ (Risk premium)	0.030 (0.035)	0.028* (0.023)
<b>Variance</b>		
$\omega$ (constant)	-0.460*** (0.036)	-0.491*** (0.068)
$\alpha_1$ (ARCH effect)	-0.171*** (0.034)	-0.172*** (0.034)
$\alpha_2$	0.091** (0.034)	0.093** (0.034)
$\beta_1$ (GARCH effect)	0.950*** (0.004)	0.947*** (0.007)
$\gamma_1$ (leverage effect)	-0.214*** (0.044)	-0.211*** (0.044)
$\gamma_2$	0.024 (0.043)	0.024 (0.042)
Log likelihood	8.841.677	8.841.947
AIC	-6.4364	-6.4366
BIC	-6.4126	-6.4128

Note 1: \*\*\*Significant at 0.1%, \*\* Significant at 1%, \*Significant at 5%

Note 2: The values in the parenthesis represent the standard errors

In the assessment of volatility models for the Vietnamese stock market, the significance of ARCH( $\alpha_1$ ) and GARCH( $\beta_1$ ) effects within both specifications of EGARCH(2,1)-M models have been robustly validated. The focal point of these models - the risk premium parameter  $\lambda$  - has yielded intriguing insights into market dynamics.

Empirical results indicate that both versions of the EGARCH(2,1)-M models show a positive risk premium, which supports the financial theory of a risk-return trade-off in the Vietnamese context. Although, while the model considering conditional standard deviation shows a positive but not statistically significant risk premium, the model using conditional variance exhibits a positive statistically significant result. This confirms the presence of a positive risk-return relationship, which means that increases in market volatility, as measured by the variance, are associated with increases in expected returns. The conditional variance used as proxy for risk is positively related to the level of return, supporting the theory of a positive risk premium on stock indices which states that the higher returns are expected for assets with

higher levels of risk. According to De Santis & Imrohorglu (De Santis & Imrohorglu, 1997), these two measures of risk (conditional standard deviation and conditional variance) have one characteristic in common: they both assume that country specific risk should be priced. This implies that the financial market is assumed to be perfectly segmented, which is the case for the Vietnamese one.

In any case, it is the model using conditional variance that can consider better the asymmetry and leptokurtosis (fat tails) in financial return distribution; thus, it is the more suitable for the empirical distributions with these characteristics that are generally observed in emerging markets like Vietnam.

## 5. Conclusions

This research, using robust symmetric and asymmetric GARCH models and considering within them three different error term distributions (Normal, Student-t and Skewed-t), provides an in-depth examination of the volatility dynamics and the risk-return relationship in the Vietnamese stock market, in particular focusing on the VN-INDEX of the Ho Chi Minh Stock Exchange. The study refers to daily data, spanning from August 2004 to August 2024. Actually, it is one of the very few paper that considers a so long period, moreover including the effects of some unexpected exogenous shocks (like Covid-19 pandemic and other recent geopolitical events) on the parameters estimates of the models used to study the relationship between volatility and expected returns, and that highlights the importance of carefully considering the choice of the error distribution within the models, as it can significantly impact the parameter estimation.

The preliminary analyses revealed significant characteristics of the VN-INDEX returns series. The descriptive statistics, QQ plot, and histogram of the returns indicated non-normality, excess kurtosis, and negative skewness, suggesting a left-skewed, peaked distribution with heavy tails. These features, typical of financial data, imply a higher likelihood of extreme negative returns compared to extreme positive returns. Further preliminary tests confirmed that the returns series are stationary and exhibit both autocorrelation and ARCH effects, thus providing a foundation for GARCH volatility modeling approach.

As mentioned, in the volatility analysis, several models from the GARCH family, including GARCH(1,1), EGARCH(1,1), GJR-GARCH(1,1), and APARCH(1,1) with three different distributional assumptions, i.e. Normal, Student-t, and Skewed-t, were implemented. The estimation results reinforced the presence of well-known stylized facts in financial time series, such as high volatility persistence, volatility clustering and leverage effects. All these findings align with international evidence.

After comparing models using Information Criteria (AIC and BIC) and performing diagnostic checks, EGARCH(2,1) model with a Skew-t error distribution revealed to be the best fit model for the dataset. This model effectively eliminated autocorrelation and ARCH effects in the standardized residuals, confirming the suitability of the Skewed-t distribution for capturing the volatile nature of the error term.

Finally, to investigate the risk-return relationship an EGARCH(2,1)-M model was employed.

This model highlights a significant positive risk premium parameter when using conditional variance as a potential explanatory variable for expected returns.

Future research could explore the impact of some macroeconomic variables (such as, among the other, GDP growth, inflation and exchange rates) on stock market volatility to provide a more holistic view of market dynamics. Another extension of this work could involve a comparative analysis between the Vietnamese stock market and other emerging markets; for example, by applying the same volatility modeling techniques across multiple emerging markets, can be identified both unique and common factors affecting them and thus could be also investigated how different regulatory environments, market structures, and economic conditions could affect both volatility and the risk-return relationship in different contexts. Finally, since different non-normal error distributions can capture asymmetries that the model alone cannot, future research should explore different non-normal distributions to improve the accuracy of the GARCH models results.

These extensions would build on the current study's foundation and could offer valuable insights for investment strategies and policy decisions in all the emerging markets like Vietnam.

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### Notes

Note 1. These prices have undergone a thorough comparison with other data sources, and no discrepancies have been detected. R software has been used to implement all the statistical tests, modelling techniques and obtain the results.

Note 2. Data available upon request in relation to all these analyses.

Note 3. Data available upon request.

Note 4. Data available upon request.